The 2nd Asian Nuclear Reaction Database Development Workshop

The systematics of (n,2n) reaction excitation function

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1. Introduction

The (n,2n) reaction is neutron multiplication reaction.

Measured data are scarce and scattered for some nuclei or energy regions.

Based on the existed experimental data, the systematics are convenient and reliable for prediction of the cross sections or other nuclear data.

Earlier works

- ✓ **Excitation function**

1961, D.W.Barr et al, 15 nuclei

1975, W.G.Davey et al, 61 nuclei

1985, Zhang Jin et al, 70 nuclei

1990, Yao Lishan et al, 98 nuclei

- ✓ **Other works are performed on the neutron energy of about 14 MeV.**

2. Formulae

- **Constant temperature evaporation model**

  + preequilibrium emission

$$\sigma_{n,2n} = \sigma_{n,2n}^{eq} + \sigma_{n,2n}^{pre}$$

- **Empirically, the preequilibrium emission can only occur ... first neut...**

Nonelastic cross section

Neutron emission cross section of compound nucleus

$$\sigma_{n,2n} = (1-\delta)\sigma_{ne}\left(\frac{\sigma_{n,M}}{\sigma_{ne}}\right)_{eq}\left(\frac{\sigma_{n,2n}}{\sigma_{n,M}}\right)_{eq} + \delta\sigma_{ne}\left(\frac{\sigma_{n,M}}{\sigma_{ne}}\right)_{pre}\left(\frac{\sigma_{n,2n}}{\sigma_{n,M}}\right)_{pre}$$

Contribution of preequilibrium emission

Equilibrium process

Preequilibrium emission process

- **Supposing that** $\left(\frac{\sigma_{n,M}}{\sigma_{ne}}\right)_{eq} \approx \left(\frac{\sigma_{n,M}}{\sigma_{ne}}\right)_{\mathrm{pre}} \approx \frac{\sigma_{n,M}}{\sigma_{ne}}$

$$\sigma_{n,2n} = \sigma_{ne}\frac{\sigma_{n,M}}{\sigma_{ne}}\left[(1-\delta)\left(\frac{\sigma_{n,2n}}{\sigma_{n,M}}\right)_{eq} + \delta\left(\frac{\sigma_{n,2n}}{\sigma_{n,M}}\right)_{pre}\right]$$

- **Taking the competition of (n,3n) reaction into account**

$$\sigma_{n,2n} = \sigma_{ne}\frac{\sigma_{n,M}}{\sigma_{ne}}\left\{(1-\delta)\left[\left(\frac{\sigma_{n,2n}}{\sigma_{n,M}}\right)_{eq} - \left(\frac{\sigma_{n,3n}}{\sigma_{n,M}}\right)_{eq}\right] + \delta\left[\left(\frac{\sigma_{n,2n}}{\sigma_{n,M}}\right)_{pre} - \left(\frac{\sigma_{n,3n}}{\sigma_{n,M}}\right)_{pre}\right]\right\}$$

3. Experimental data fitting

- The experimental data were taken from EXFOR and other references.
- 200 nuclei, A=2-241.
- 120 nuclei were selected for fitting.
- Based on the systematic equations, using the nonlinear least squares method, the local parameters have been obtained are shown in next table.

Table 1. The results of fitting

Z-El-A	*(N-Z)/A*	*T*	ΔT	$\sigma_{n,M}/\sigma_{n,e}$	$\Delta\sigma_{n,M}/\sigma_{n,e}$	**Data points**	χ^2
7-N-14	0	2.2279	0.1162	0.02316	0.00116	52	2.22619
9-F-19	0.05263	3.2705	0.1654	0.15845	0.0047	79	2.52151
11-Na-23	0.04348	1.9116	0.0862	0.18915	0.00831	48	4.44015
15-P-31	0.03226	1.8166	0.2605	0.08007	0.00884	8	1.18698
16-S-32	0	0.7328	0.2294	0.00172	0.00074	8	1.38801
19-K-39	0.02564	3.0803	0.5604	0.05203	0.01078	19	4.15205
20-Ca-40	0	0.8824	0.3418	0.04075	0.01874	7	2.84086
20-Ca-48	0.16667	1.3399	0.2544	0.89805	0.06649	13	0.616787
21-Sc-45	0.06667	1.8023	0.0593	0.52162	0.01572	53	2.05532
22-Ti-46	0.04348	1.8718	0.092	0.28312	0.01645	37	2.80857
23-V-51	0.09804	1.1491	0.1031	0.73474	0.01382	7	0.395842
24-Cr-50	0.04	1.7427	0.1459	0.09682	0.01053	70	10.2234
24-Cr-52	0.07692	1.3409	0.0319	0.61914	0.01384	63	4.72366
25-Mn-55	0.09091	1.1654	0.035	0.74655	0.00782	121	1.45862
26-Fe-54	0.03704	2.1004	0.2819	0.10228	0.01662	30	12.599
26-Fe-56	0.07143	1.3566	0.1083	0.57554	0.04931	10	1.21987

27-Co-59	0.08475	1.332	0.0184	0.75792	0.00637	168	1.99724
28-Ni-58	0.03448	1.8189	0.0334	0.07994	0.00171	239	5.25956
29-Cu-63	0.07937	2.0622	0.0402	0.74043	0.01104	180	13.4766
29-Cu-65	0.10769	1.459	0.0121	0.87743	0.00254	202	4.63472
30-Zn-64	0.0625	2.279	0.1588	0.29871	0.02633	69	6.73927
30-Zn-66	0.09091	1.354	0.0215	0.76999	0.00878	32	0.39661
30-Zn-70	0.14286	1.8208	0.465	1.26898	0.2888	6	2.35775
31-Ga-69	0.10145	1.4258	0.156	0.89285	0.05512	30	1.94808
31-Ga-71	0.12676	1.2756	0.4422	0.84936	0.1381	9	6.52399
32-Ge-70	0.08571	1.3354	0.0736	0.73445	0.03222	48	3.27118
32-Ge-72	0.11111	1.1578	0.3679	0.76193	0.1423	4	1.04694
32-Ge-76	0.15789	1.0343	0.1295	0.87828	0.02694	38	2.53885
33-As-75	0.12	1.237	0.0512	0.88583	0.01845	42	1.37546
34-Se-74	0.08108	1.2562	0.077	0.54148	0.02163	32	5.91725
34-Se-76	0.10526	1.32	0.122	0.85699	0.06255	29	2.22596
34-Se-78	0.12821	1.4299	0.0936	0.9029	0.0623	9	0.780916
34-Se-80	0.15	1.108	0.0234	0.80034	0.01627	10	0.160675
34-Se-82	0.17073	1.4778	0.1046	1.07462	0.04737	27	4.37264
35-Br-79	0.11392	1.2241	0.0628	0.83716	0.02441	40	1.45509
35-Br-81	0.1358	1.318	0.1114	0.88206	0.03733	18	0.471761
37-Rb-85	0.12941	1.268	0.0515	0.87084	0.01942	49	1.78063

37-Rb-87	0.14943	1.2911	0.1151	0.95031	0.03504	31	1.42041
38-Sr-84	0.09524	1.5714	0.159	0.83031	0.0967	14	0.607304
38-Sr-86	0.11628	1.297	0.1193	0.91221	0.0723	12	0.732731
39-Y-89	0.1236	1.1285	0.0256	0.89204	0.01343	141	3.23874
40-Zr-90	0.11111	1.2142	0.022	0.85641	0.01279	118	3.37421
41-Nb-93	0.11828	1.374	0.0345	0.92035	0.01355	31	1.03499
42-Mo-92	0.08696	2.1698	0.2675	0.84108	0.1084	10	2.48585
42-Mo-100	0.16	1.1458	0.0897	0.93181	0.01236	86	2.77571
44-Ru-96	0.08333	1.1892	0.0436	0.64274	0.0124	9	0.174075
44-Ru-98	0.10204	1.0283	0.1833	0.81747	0.05408	5	1.01058
44-Ru-104	0.15385	1.2069	0.154	1.064	0.03278	12	1.40399
45-Rh-103	0.12621	1.4211	0.0416	0.91704	0.01612	26	1.86819
46-Pd-102	0.09804	1.0649	0.0681	0.71865	0.01884	42	1.10991
46-Pd-110	0.16364	1.0152	0.1875	1.06321	0.02678	18	1.17411
48-Cd-106	0.09434	1.3175	0.1547	0.67701	0.05348	17	2.19159
48-Cd-116	0.17241	1.2882	0.0804	0.99835	0.02068	17	1.18494
49-In-113	0.13274	1.4587	0.1667	1.02774	0.05159	41	0.856408
49-In-115	0.14783	1.245	0.1215	0.99372	0.02913	18	0.986357
50-Sn-112	0.10714	1.2505	0.1235	1.09464	0.06195	27	1.42958
50-Sn-114	0.12281	1.3142	0.1449	0.83387	0.05228	13	0.92338
51-Sb-121	0.15702	1.3636	0.0846	0.96479	0.02021	34	1.10706

51-Sb-123	0.17073	1.2786	0.1488	0.86802	0.02839	18	1.52888
52-Te-120	0.13333	1.6906	0.1663	0.95635	0.0645	11	0.172672
52-Te-122	0.14754	1.3313	0.3216	1.00086	0.101	9	0.518551
52-Te-128	0.1875	1.4751	0.069	1.05845	0.01498	11	0.285467
52-Te-130	0.2	1.3433	0.0838	1.02271	0.01817	20	2.31384
53-I-127	0.16535	1.2203	0.0177	0.99049	0.01745	28	1.77864
55-Cs-133	0.17293	1.1757	0.2365	0.83903	0.03467	16	1.00313
56-Ba-130	0.13846	1.1259	0.0771	0.95846	0.02898	6	0.105193
56-Ba-132	0.15152	1.0015	0.1313	0.95481	0.03212	9	0.384762
56-Ba-134	0.16418	0.771	0.0995	0.94198	0.01135	17	0.73186
58-Ce-136	0.14706	1.4309	0.2566	0.89684	0.08424	20	0.793848
58-Ce-140	0.17143	1.2044	0.0646	0.99866	0.01345	63	1.0458
59-Pr-141	0.16312	1.277	0.0853	0.96159	0.0206	61	0.915619
60-Nd-142	0.15493	1.1181	0.042	0.9563	0.01704	33	0.807607
60-Nd-144	0.16667	0.8618	0.0644	0.80471	0.04032	12	3.3384
60-Nd-146	0.17808	0.9088	0.0379	0.87805	0.02856	11	0.995522
60-Nd-148	0.18919	1.0348	0.0561	0.98497	0.03395	16	4.16039
60-Nd-150	0.2	1.1459	0.0282	1.00034	0.01312	36	1.42895
62-Sm-144	0.13889	1.4481	0.0921	0.9806	0.03866	34	0.757469
62-Sm-148	0.16216	0.9326	0.0444	0.83558	0.03407	10	1.06571
62-Sm-150	0.17333	1.1408	0.0284	0.90729	0.01756	26	1.52132

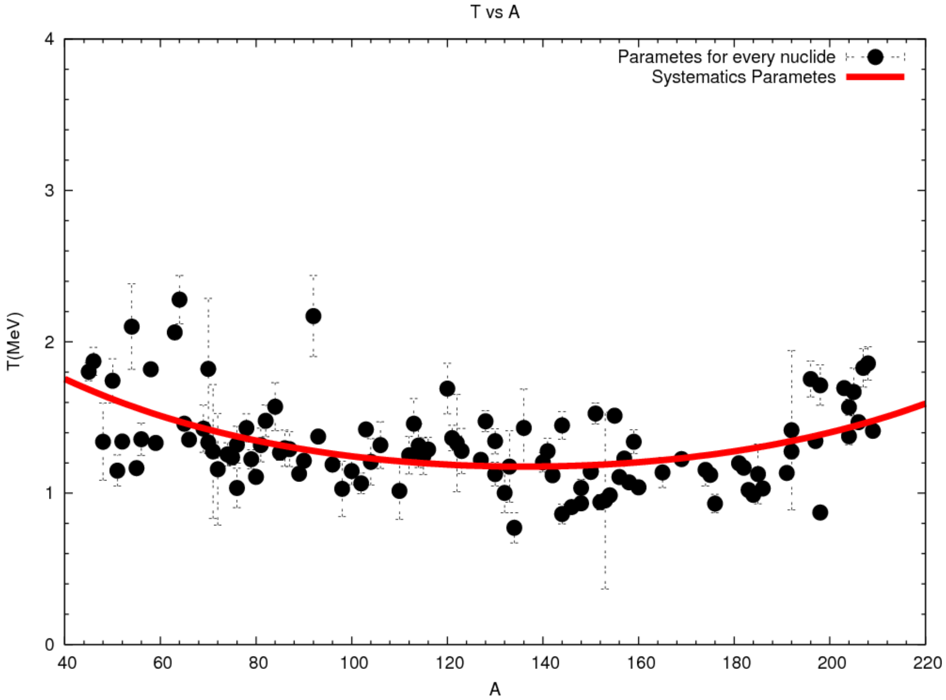
62-Sm-152	0.18421	0.941	0.0506	0.86236	0.04009	10	1.4651
62-Sm-154	0.19481	0.9855	0.0318	0.94387	0.02025	14	0.676829
63-Eu-151	0.16556	1.5262	0.0701	0.91839	0.02848	23	1.48006
63-Eu-153	0.17647	0.9518	0.5834	0.86233	0.0481	19	1.70945
64-Gd-155	0.17419	1.5118	0.0426	0.87879	0.01867	17	0.450916
64-Gd-156	0.17949	1.1066	0.0258	0.88125	0.01466	12	0.200117
64-Gd-157	0.18471	1.2286	0.0233	0.85046	0.01096	17	0.232949
64-Gd-158	0.18987	1.0712	0.0428	0.88511	0.02434	14	0.745737
64-Gd-160	0.2	1.0386	0.0444	0.94403	0.01818	30	1.49973
65-Tb-159	0.18239	1.3396	0.0782	0.97603	0.01637	14	0.658835
67-Ho-165	0.18788	1.1368	0.1002	0.83032	0.01876	14	0.590646
69-Tm-169	0.18343	1.2255	0.0214	0.96836	0.00821	80	2.35519
71-Lu-175	0.18857	1.1213	0.0295	0.8953	0.0158	41	2.61883
72-Hf-174	0.17241	1.1537	0.1067	0.90028	0.03473	6	0.736337
72-Hf-176	0.18182	0.9311	0.0617	0.96279	0.01858	14	1.22371
73-Ta-181	0.19337	1.1972	0.0324	0.94012	0.01511	25	1.47848
74-W-182	0.18681	1.168	0.0214	0.96663	0.00969	22	0.429228
74-W-183	0.19126	1.0212	0.0108	0.82275	0.00604	14	0.082205
74-W-184	0.19565	0.9887	0.0227	0.8846	0.01792	15	0.527287
74-W-186	0.2043	1.0312	0.0185	0.92309	0.01103	31	0.651484
75-Re-185	0.18919	1.1265	0.1973	0.99564	0.02468	11	1.24692
76-Os-192	0.20833	1.2757	0.049	1.02707	0.01161	30	0.832582

77-Ir-191	0.19372	1.1332	0.0247	0.99008	0.01077	14	0.393489
78-Pt-192	0.1875	1.4158	0.5268	0.91929	0.09799	13	0.946882
78-Pt-198	0.21212	0.872	0.0304	0.85738	0.00879	13	0.453537
79-Au-197	0.19797	1.3445	0.0177	0.96985	0.00511	179	3.10207
80-Hg-196	0.18367	1.754	0.1187	1.16909	0.05882	15	0.689333
80-Hg-198	0.19192	1.7128	0.1332	0.99901	0.04024	19	1.11518
80-Hg-204	0.21569	1.3761	0.0551	0.93264	0.01395	30	0.974072
81-Tl-203	0.20197	1.6941	0.036	0.93094	0.02038	36	6.15124
81-Tl-205	0.20976	1.6692	0.1576	0.92367	0.07414	13	3.09419
82-Pb-204	0.19608	1.5685	0.0528	1.01658	0.01358	55	2.76937
82-Pb-206	0.20388	1.469	0.0445	0.90832	0.02393	14	0.405933
82-Pb-207	0.20773	1.8278	0.1278	0.88194	0.03687	17	1.56512
82-Pb-208	0.21154	1.8569	0.1112	0.9959	0.04539	17	1.60393
83-Bi-209	0.20574	1.4122	0.0426	0.9826	0.01325	31	1.41212
90-Th-232	0.22414	0.8464	0.013	0.92665	0.01275	50	2.70061
92-U-235	0.21702	1.1843	0.0793	0.31674	0.01292	16	1.15532
92-U-238	0.22689	0.7704	0.0077	0.59795	0.00399	130	19.8083
94-Pu-239	0.21339	1.0246	0.0691	0.1204	0.00547	27	5.87943
95-Am-241	0.21162	1.3109	0.0364	0.10267	0.00158	33	1.77901

4. Parameter correlations

- ✓ **On the basis of the parameters of every nuclei within 45≤A≤210 (in table 1), the correlations between the parameters and some quantity of the target can be expressed as simple functions.**

The correlations between T and A



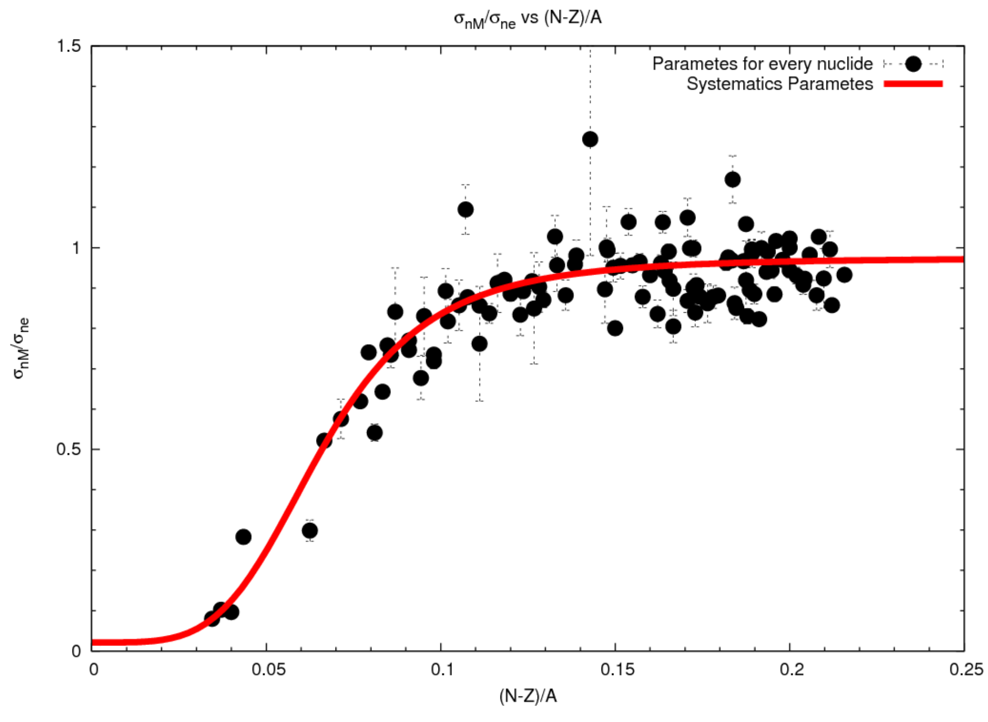
$$T = exp(a \times A^2 - b \times A + c)$$

$$a = 4.323 \times 10^{-5}$$

$$b = 0.01178$$

$$c = 0.9641$$

The correlations between $\sigma_{n,M}/\sigma_{n,e}$ and (N-Z)/A



$$\frac{\sigma_{n,M}}{\sigma_{ne}} = a - b\Big/\left\{1+\left[\frac{(\mathrm{N}-\mathrm{Z})}{c\times \mathrm{A}}\right]^{d}\right\}$$

$$a = 0.974$$

$$b = 0.953$$

$$c = 0.066$$

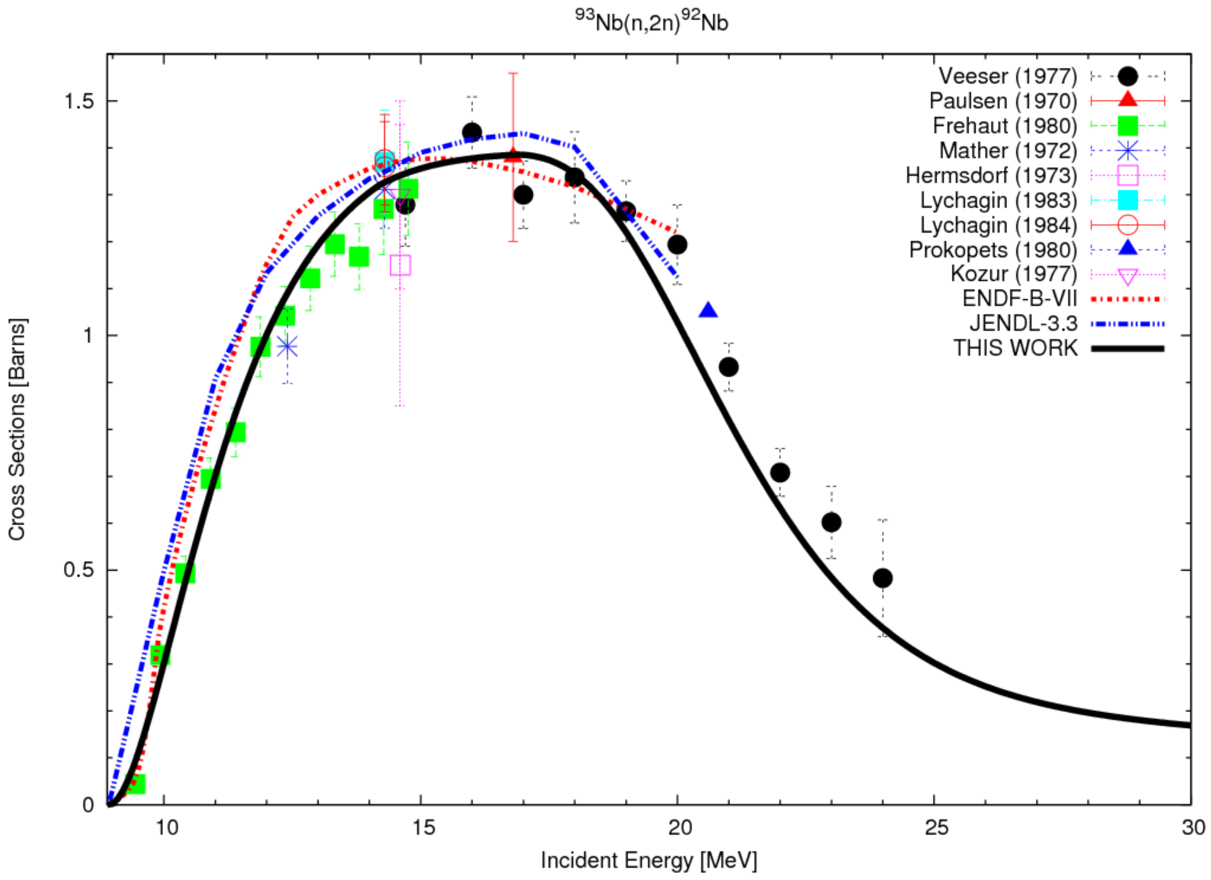
$$d = 4.225$$

5. Results and discussion

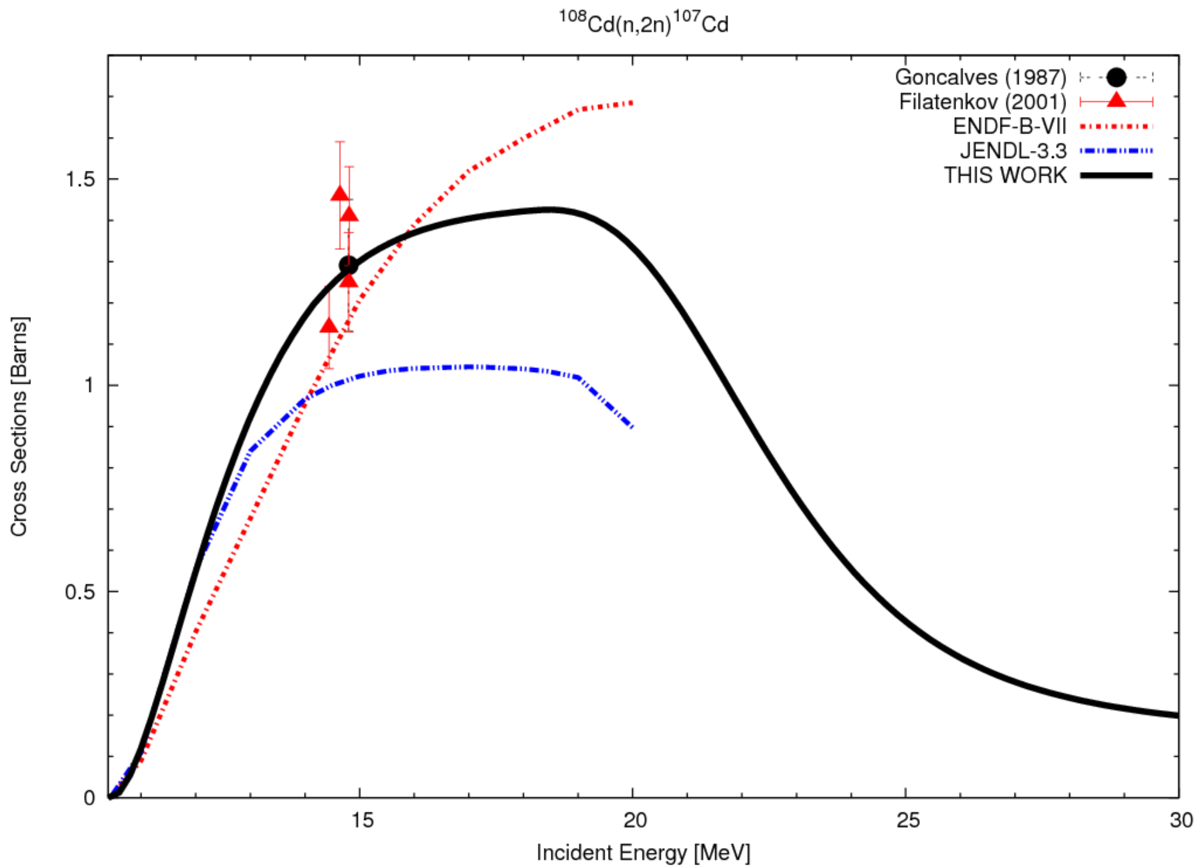
Using the regional parameters, the (n,2n) cross sections of about 290 nuclei have been calculated.

Some typical results are shown in next figures.

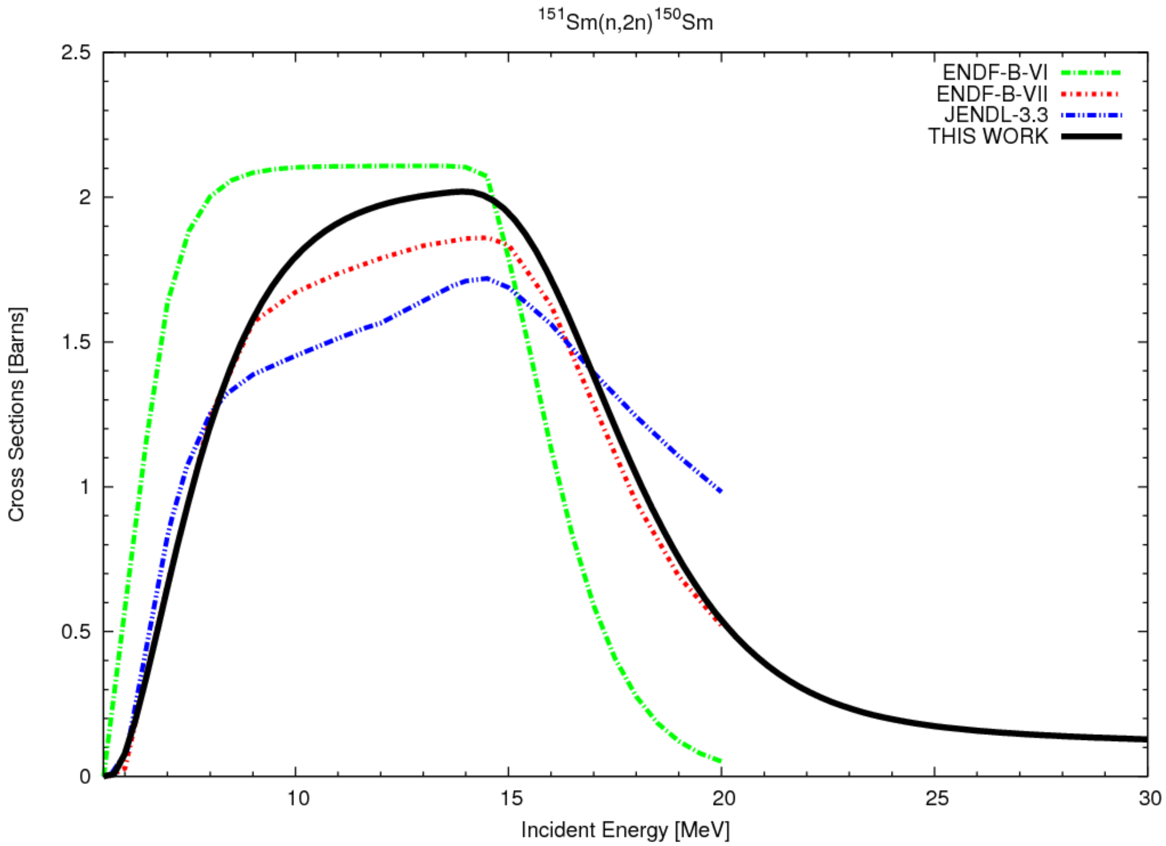
Measured data are abundant (108 nuclei)



Measured data are scarce (42 nuclei)



No measured data (142 nuclei)



Conclusions

The results indicate that the predicted cross sections are consistent with the experiment and evaluated data.

Hence, more accurate systematics prediction for unmeasured nucleus or energy range may be provided.

"The Systematics of (n,2n) Reaction Excitation Function",
Ji-Min Wang, Xi Tao, Xiao-Long Huang and You-Xiang Zhuang.
Nuclear Inst. and Methods in Physics Research, B, 268(2010)13.
PP. 2221-2227.

Thank you !

Nonelastic cross section

$$\sqrt{\sigma_{ne}/\pi} = 0.2317 + 0.1128A^{1/3} \qquad E_n \le 14.2MeV$$

$$\sqrt{\sigma_{ne}/\pi} = 0.2317 - 0.2189\ln\frac{E_n}{14.2} + 0.04777\ln\left(\frac{E_n}{14.2}\right)^2 + \left(0.1128 + 0.03263\ln\frac{E_n}{14.2}\right)A^{1/3}$$

$$E_n > 14.2MeV$$

Neutron emission cross section of compound nucleus

$$\sigma_{n,M} = \sigma_{n,n'} + \sigma_{n,2n} + \sigma_{n,3n} + \ldots\ldots$$

Contribution of preequilibrium emission

$$\delta = 1 - \exp\{-G[A(x) + A(y)]\}$$

$$G = \frac{81.4}{KA^{1/3}}$$

$$A(\xi) = (5 - 3\xi^2)\xi^3 / (1 - \xi^2)^2 \qquad \xi = x \ \ or \ \ y$$

$$x = \frac{E_n}{E_n + B}$$

$$y = \begin{cases} (E_n - V_c)/(E_n + B) & E_n > V_c \\ 0 & E_n \le V_c \end{cases}$$

Equilibrium process

Using the constant temperature type level density

$$\rho_B \propto e^{-\frac{E_b}{T}}$$

$$\left(\frac{\sigma_{n,2n}}{\sigma_{n,M}}\right)_{EQ} = 1 - \left(1 + \frac{E_n - E_{th}^{n,2n}}{T}\right)\exp\left(-\frac{E_n - E_{th}^{n,2n}}{T}\right)$$

$$E_n < E_{th}^{n,2n} \qquad \frac{E_n - E_{th}^{n,2n}}{T} = 0$$

Preequilibrium emission process

Exciton model + some approximations

$$\delta\left(\frac{\sigma_{n,2n}}{\sigma_{n,M}}\right)_{pre} = \{\delta(1-\eta^2)+\eta^2(1-\delta)G[A(x)+A(y)]\} - (1-\delta)G[F(x,\eta)+F(y,\eta)]$$

$$\eta = \frac{E_{th}^{n,2n}}{E_n}$$

$$F(\xi,\eta) = \frac{(\xi\eta)^2}{[1-(\xi\eta)^2]^3}\{\xi[15-10(\xi\eta)^2+3(\xi\eta)^4]-2\xi\eta[5-(\xi\eta)^2]\}$$

$$\xi = x \quad or \quad y$$