Memo for 2-body decay 9-Feb-2010

Annex to Memo CP-D/611

1 Relative energy (non-relativistic)

We define $E_{\rm rel}$ in the c.m. frame of the 2-body system as in,

$$E_{\rm rel} = T_1 + T_2 \quad (1)$$
$$= \frac{p^2}{2m_1} + \frac{p^2}{2m_2} \quad (2)$$
$$= \frac{p^2}{2\mu}, \quad (3)$$

where (m_1, m_2) and (T_1, T_2) are masses and kinetic energies of these particles, and μ stands for their reduced mass. The momentum in the c.m. frame for two particles have the same absolute value p.

In the case that

$$a + b \rightarrow c + d,$$

The Q value is defined as,

$$Q = (M_a + M_b)c^2 - (M_c + M_d)c^2 \quad (4)$$
$$= (T_c + T_d) - (T_a + T_b) \quad (5)$$
$$= E_{\rm rel} - E_{cm} \quad (6)$$

where $E_{\rm rel} = T_c + T_d$ and $E_{cm} = T_a + T_b$ in the c.m. frame of these two particles. (This definition of Q is common for non-relativistic and relativistic cases.)

2 E_{lab} and E_{cm} (non-relativistic)

If 'b' is the beam and 'a' is the target in the laboratory frame,

$$E_{cm} = \frac{p^2}{2\mu_{ab}} \quad (7)$$
$$= \frac{1}{2}\mu_{ab}v_b^2 \quad (8)$$
$$= \frac{M_a}{M_a + M_b}E_{Lab} \quad (9)$$

3 Relative energy (relativistic)

Invariant mass, M, of the intermediate state before decay into two particles (particle 1 and particle2), can be written as,

$$M^2 = (p_1 + p_2)^2 = (E_1 + E_2)^2 - (\vec{p_1} + \vec{p_2})^2 \quad (10)$$

Here, we use c=1 notation.

The relative energy between particle 1 and 2 is defined as,

$$E_{\rm rel} = M - (m_1 + m_2) \quad (11)$$
$$= \sqrt{(E_1 + E_2)^2 - (\vec{p_1} + \vec{p_2})^2} - (m_1 + m_2) \quad (12)$$

In the center of mass frame of particle 1 and 2,

$$\vec{p_1} + \vec{p_2} = \vec{0}.$$

Hence,

$$E_{\rm rel} = E_1 + E_2 - (m_1 + m_2) = T_1 + T_2. \tag{13}$$

This is the same definition as in the non-relativistic case.